

Part A: Multiple Choice (20 marks, 1 mark each)

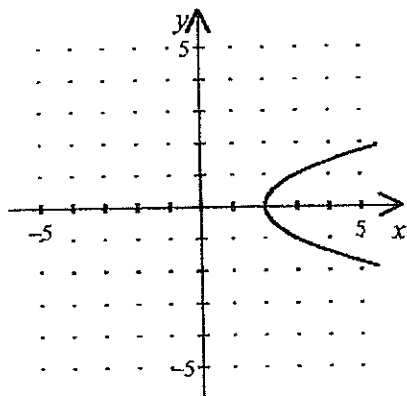
Circle the correct answer.

1. The relation that is not a function is

- A. $\{(2, 5), (-2, 6), (3, 6), (6, -2)\}$
- C. $\{(5, 2), (-2, 5), (-5, 2), (2, 5)\}$

- B.** $\{(4, 3), (4, 5), (6, -1), (3, 4)\}$
- D. $\{(1, 2), (2, 3), (3, 4), (4, 5)\}$

2. Find the domain and range of the relation.

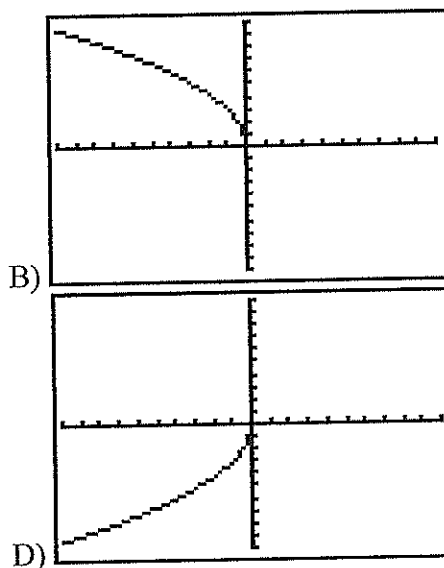
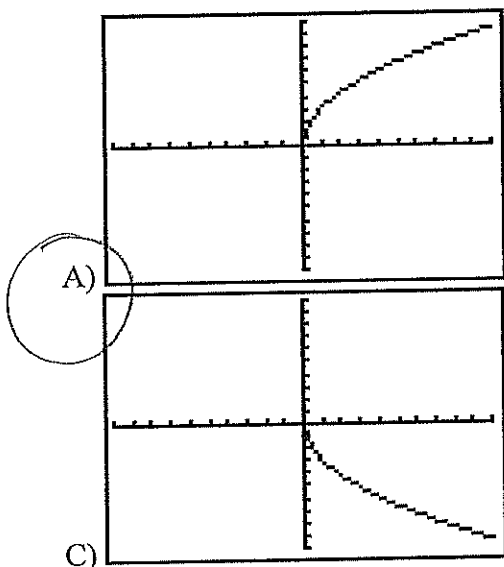


- A.** Domain: $x \geq 2$, range: the set of real numbers
- B. Domain: the set of real numbers, range: $x \geq 2$
- C. Domain: the set of real numbers, range: $y \geq 2$
- D. Domain: $y \geq 2$, range: the set of real numbers

3. For the function $y = 2\sqrt{9-x}$, what is the domain?

- A) $x \geq 9$
- B) $x \geq 18$
- C) $x \leq 18$
- D) $x \leq 9$**
- E) $x \leq 6$

4. The graph that best represents $y = 3\sqrt{x}$ is

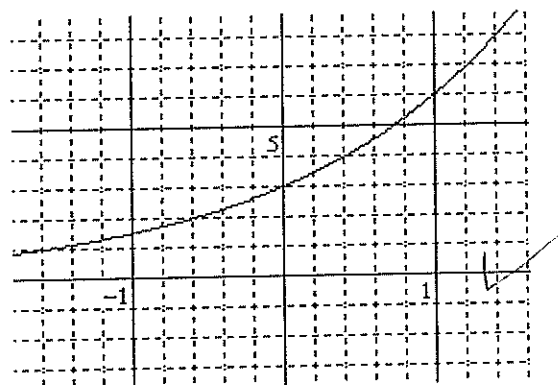


5. State the restrictions for the expression: $\frac{5x^2 - 1}{5x^2 - 3x}$

- A) $x \neq 0, \frac{5}{3}$**
- B) $x = -1, 0, \frac{5}{3}$
- C) $x = -1, \frac{5}{3}$
- D) $x = \frac{5}{3}$

6. Which equation best represents the following graph?

- A) $f(x) = 2(3)^x$ B) $f(x) = 3\left(\frac{1}{2}\right)^x$
 C) $f(x) = 2\left(\frac{1}{2}\right)^x$ **D) $f(x) = 3(2)^x$**



7. The y-intercept of $f(x) = 3(2)^{2x-1}$ is

- A) 3 B) 1 **C) $\frac{3}{2}$** D) 0

8. \$2000 is invested at 5% per year compounded semi-annually for 3 years. The interest rate for each conversion period is

- A) 0.05 B) 0.10 **C) 0.025** D) 0.0025

9. $64^{\frac{1}{2}}$ is equal to

- A) 32 B) 4 **C) 8** D) 16

10. $\left(\frac{25}{64}\right)^{-\frac{3}{2}}$ is equal to

- A) $\frac{115}{511}$ B) $\frac{125}{155}$ C) $\frac{155}{125}$ **D) $\frac{511}{115}$**

11. Solve for x , exactly when $2x^2 - 120 = 0$

- A) $\pm 4\sqrt{30}$ B) $\pm 4\sqrt{15}$ C) $\pm 2\sqrt{30}$ **D) $\pm 2\sqrt{15}$**

12. If $f(\theta) = -2\sin(6(\theta + 30^\circ))$, determine the phase shift.

- A) -180° B) 30° C) 180° **D) -30°**

13. Which of the following statements is true for $y = 3(10x - 45)^2 - 749$?

- A) The parabola opens up and has two x-intercepts.**
 B) The parabola opens up and has one x-intercept.
 C) The parabola opens down and has no x-intercepts.
 D) The parabola opens down and has two x-intercepts.

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14. State the value of the discriminant, D , and the number of roots for $7x^2 + 12x + 6 = 0$.
- A) ~~$D = 312, n = 2$~~ B) $D = 24, n = 2$ C) $D = 312, n = 1$ D) $D = -24, n = 0$ ✓

$$(12)^2 - 4(7)(6) = -24$$

15. The fifth term of the geometric sequence 2, 12, 72, ... is
- A) 5184 B) 2592 C) 1296 D) 10368 ✓

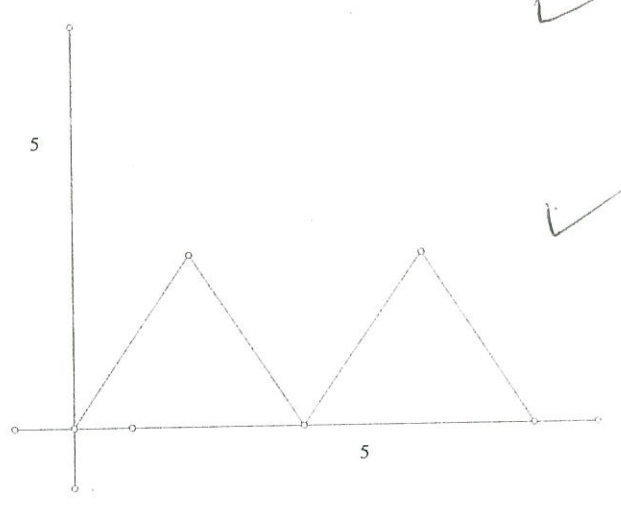
16. Solve for θ when $\sin \theta = -\frac{1}{\sqrt{2}}$ for the domain $0^\circ \leq \theta \leq 360^\circ$.
- A) 135° and 225° B) 225° and 315° C) 45° and 135° D) 135° and 315° ✓



17. In what way does the graph of $y = \sin \theta$ differ from the graph of $y = \cos \theta$?
- A) different period B) different amplitude C) different zeros D) different maximums ✓

For questions 18-20, refer to the graph on the right:

18. The graph has a period of
- A) 4 units B) 8 units
C) 2 units D) 6 units
19. The graph has an amplitude of
- A) 3 units B) 4 units
C) 1.5 units D) 6 units
20. The equation of the axis of the curve is
- A) $x = 3$ B) $y = 3$
C) $y = 1.5$ D) $x = 1.5$



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PART B: Full Solutions (75 marks)

SHOW ALL WORK!

1. Name the transformations that need to be applied to $y = f(x)$ to arrive at $y = -3f(2(x+6)) - 5$: (5 marks)

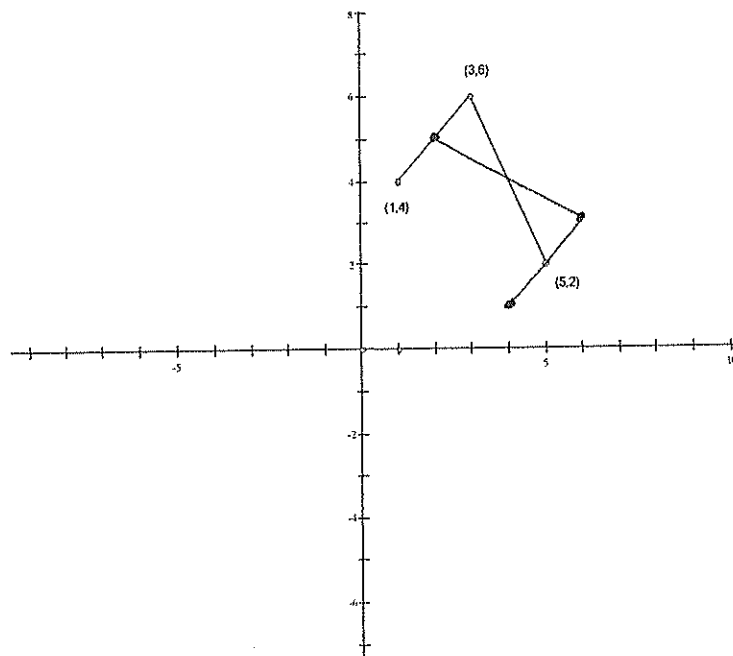
- $2x+6$ $2(x+3)$
 1. reflection in x-axis
 2. vertical stretch factor 3
 3. horizontal compression factor $\frac{1}{2}$
 4. horizontal translation Left
 5. vertical translation down 5

2. Determine the coordinates of the vertex of the following parabola, by completing the square. (no decimals) (3 marks)

$$\begin{aligned}
 y &= -4x^2 - 40x + 24 \\
 &= -4(x^2 + 10x + 25 - 25) + 24 \\
 &= -4(x^2 + 10x + 25) + 100 + 24 \\
 &= -4(x + 5)^2 + 124
 \end{aligned}$$

Vertex $(-5, 124)$

3. The graph of a relation is shown:
 a) Graph $y = f^{-1}(x)$. (2 marks)



- $(4, 1)$
 $(6, 3)$
 $(2, 5)$

- b) State the domain and range of $y = f^{-1}(x)$, in set notation. (2 marks)

Domain: $\{x \mid 2 \leq x \leq 6\}$
 Range: $\{y \mid 1 \leq y \leq 5\}$

4. Simplify **and** state the restrictions.

(4, 5 marks)

a) $\frac{2x^2 - 11x - 6}{3x^3 - 18x^2} \div \frac{4x^2 - 1}{2x^2 - x}$

$$= \frac{(2x+1)(x-6)}{3x^2(x-6)} \times \frac{x(2x-1)}{(2x-1)(2x+1)}$$

$$= \frac{\cancel{(2x+1)}\cancel{(x-6)}}{3x^2\cancel{(x-6)}} \times \frac{\cancel{x}\cancel{(2x-1)}}{\cancel{(2x-1)}\cancel{(2x+1)}}$$

$$= \frac{1}{3x}$$

$$x \neq 0, 6, -\frac{1}{2}, \frac{1}{2}$$

b) $\frac{4x}{x+7} + \frac{x+1}{x^2+5x-14}$

$$= \frac{4x}{(x+7)} + \frac{(x+1)}{(x+7)(x-2)}$$

$$= \frac{4x(x-2) + (x+1)}{(x+7)(x-2)}$$

$$= \frac{4x^2 - 8x + x + 1}{(x+7)(x-2)}$$

$$= \frac{4x^2 - 7x + 1}{(x+7)(x-2)}$$

$$x \neq -7, 2$$

5. Prove the following identity.

(4 marks)

$$\cot \theta + \tan \theta = \sec \theta \csc \theta$$

L.S.

$$= \frac{\cos \theta}{\sin \theta} + \frac{\sin \theta}{\cos \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta \cos \theta}$$

$$= \frac{1}{\sin \theta} \times \frac{1}{\cos \theta}$$

$$= \csc \theta \sec \theta$$

$$\therefore \text{L.S.} = \text{R.S.}$$

6. Simplify $\sqrt{20} - 3\sqrt{45} + \sqrt{40}$

(2 marks)

$$= \sqrt{4}\sqrt{5} - 3\sqrt{9}\sqrt{5} + \sqrt{4}\sqrt{10}$$

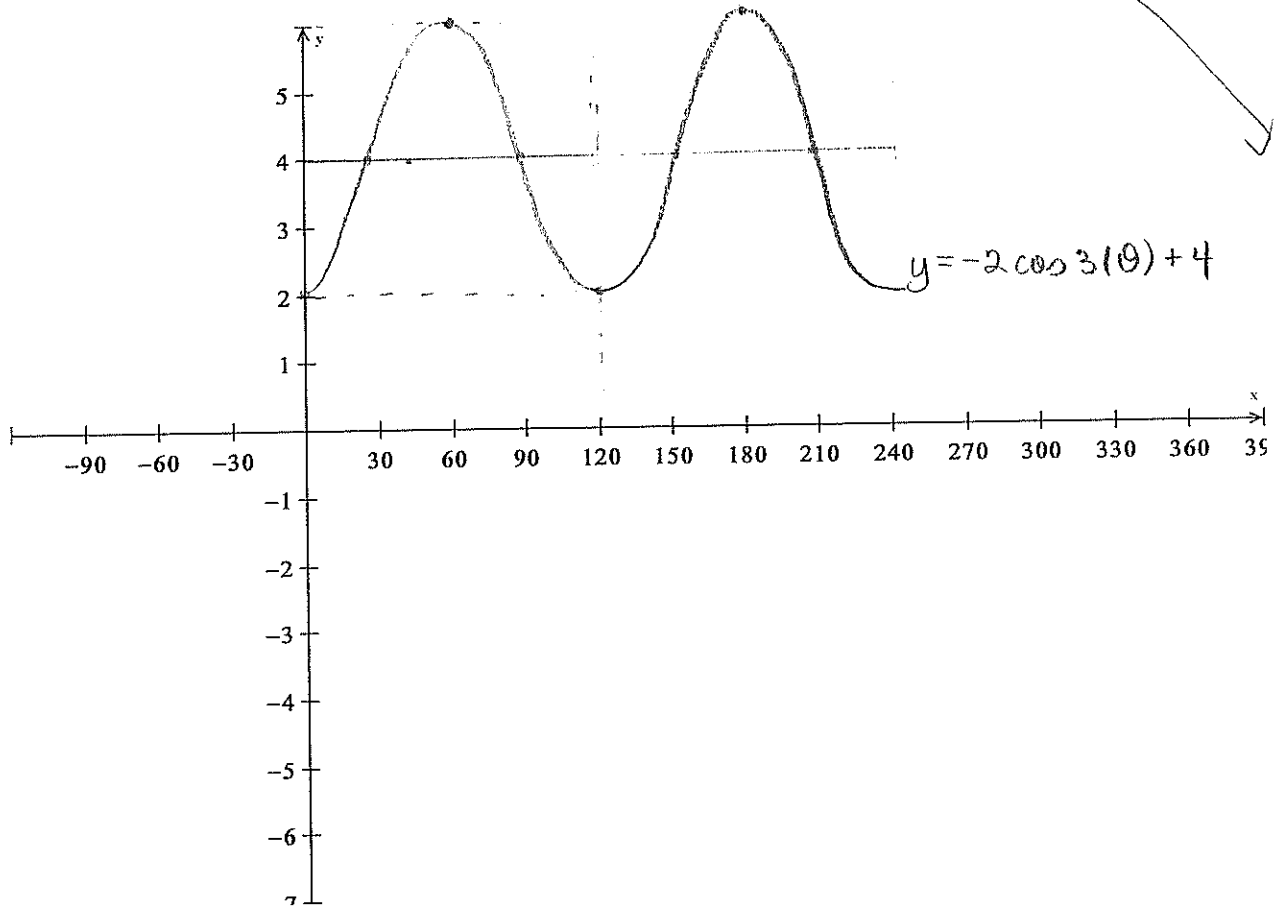
$$= 2\sqrt{5} - 9\sqrt{5} + 2\sqrt{10}$$

$$= -7\sqrt{5} + 2\sqrt{10}$$

$$p = \frac{360}{3}$$

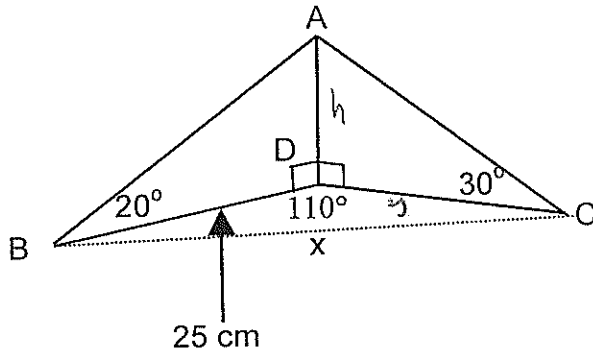
$$= 120$$

7. Sketch the graph of $f(\theta) = -2 \cos 3\theta + 4$ for the domain $0^\circ \leq \theta \leq 360^\circ$. (4 marks)



8. Determine the distance between B and C .

(5 marks)



$$\tan 20^\circ = \frac{h}{25}$$

$$h = 9.1 \text{ cm}$$

$$\tan 30^\circ = \frac{9.1}{y}$$

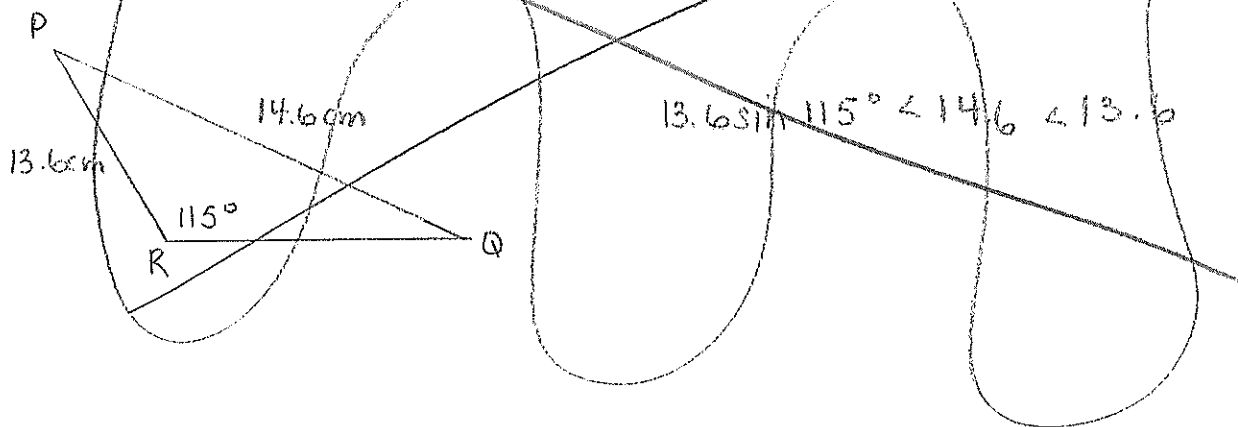
$$y = 15.8$$

$$x^2 = 25^2 + 15.8^2 - 2(25)(15.8) \cos 110^\circ$$

$$= 1144.84$$

$$x \doteq 33.8 \text{ cm}$$

9. In $\triangle PQR$, $\angle R = 115^\circ$, $q = 13.6$ cm, and $r = 14.6$ cm. Draw and label a diagram then determine the number of solutions (i.e. number of triangles). **DO NOT SOLVE.** (3 marks)



10. Stephan has \$4000 to invest today. How much will his investment be worth in 10 years if the bank offers her a rate of 3.75% compounded semi-annually? (2marks)

$$A = 4000 \left(1 + \frac{0.0375}{2} \right)^{20}$$

$$= \$5799.79$$

\therefore the investment is worth \$5799.79.

11. Simone is saving to purchase a new home. She needs \$37,000 in five years for her down payment.
- If the bank pays 4.1% compounded monthly, how does she need to invest monthly? (4 marks)
 - How much interest will she earn? (2 marks)

$$a) R = \frac{37000 \left(\frac{0.041}{12} \right)}{\left(\left(1 + \frac{0.041}{12} \right)^{60} - 1 \right)}$$

$$= \frac{126.42}{0.227}$$

$$= \$556.67$$

\therefore she must invest \$3341.41 monthly.

$$b) \text{ Total invested} = 33414.98$$

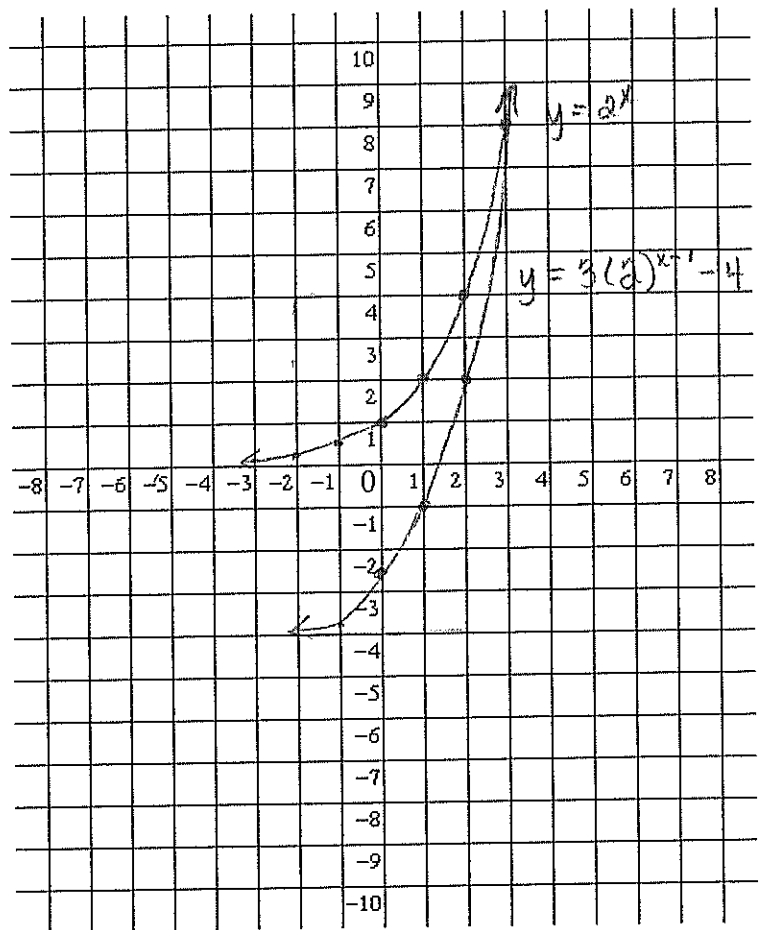
$$556.67 \times 60 = 33400.20$$

$$\text{Interest} = 37000 - 33414.98$$

$$= \$3585.02$$

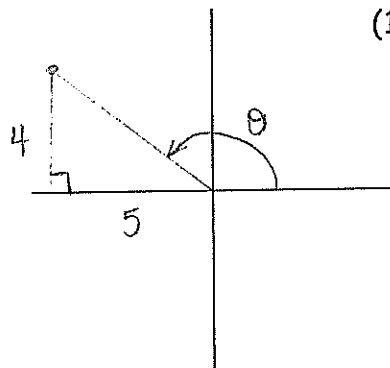
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12. **Graph** the function $f(x) = 2^x$. On the same set of axes, **graph** the function $g(x) = 3(2)^{x-1} - 4$.
Be sure to include all relevant information. (4 marks)



13. Given the point $(-5, 4)$ on the terminal arm of an angle θ in standard position.

a) Sketch and label the principal angle. (1 mark)



b) Determine the three primary trigonometric ratios for θ , to 4 decimal places. (3 marks)

$$\begin{aligned} c^2 &= 5^2 + 4^2 \\ &= 41 \\ c &= \sqrt{41} \end{aligned}$$

$$\begin{aligned} \sin \theta &= \frac{4}{\sqrt{41}} \\ &= 0.6247 \end{aligned}$$

$$\begin{aligned} \cos \theta &= -\frac{5}{\sqrt{41}} \\ &= -0.7809 \end{aligned}$$

$$\tan \theta = -\frac{4}{5}$$

c) Determine the value of the principal angle to the nearest degree. (2 marks)

$$\begin{aligned} \cos \theta &= -5/\sqrt{41} \\ \theta &= 141.3^\circ \end{aligned}$$

14. Using special triangles, determine the exact values of each of the following. (2 marks)

a) $\sin(315^\circ) = -\frac{1}{\sqrt{2}}$

b) $\tan(120^\circ) = -\sqrt{3}$

15. Determine all values of θ , to the nearest tenth of a degree, $0^\circ \leq \theta \leq 360^\circ$. (4 marks)

a) $\cos \theta = -0.40$

$\theta_1 = 113.6^\circ$

$\theta_2 = 246.4^\circ$

b) $\tan \theta = 3.4874$

$\theta_1 = 74.0^\circ$

$\theta_2 = 254.0^\circ$

16. Determine the sum of the following series, $123 + 118 + 113 + \dots - 122$. (4 marks)

$t_n = a + (n-1)d$
 $-122 = 123 + (n-1)(-5)$

$-250 = -5n$

$50 = n$

$S_{50} = \frac{50}{2} (2(123) + 49(-5))$

$= 25(246 - 245)$

$= 25$

17. For the sequence $-0.5, 1.5, -4.5, \dots$ Determine the value of the 15th term. (2 marks)

$t_{15} = -0.5(-3)^{14}$

$= -2391484.5$

18. A culture has 750 bacteria. The number of bacteria doubles every 5 hours.

a) Write an equation to model the number of bacteria over time. (2 marks)

$N = 750(2)^{t/5}$

b) How many bacteria are in the culture after 12 hours? (2 marks)

$N = 750(2)^{12/5}$

$= 3958.52$

\therefore there are 3958.52 bacteria after 12 hours

19. Simplify $\left[\frac{x^{-4}}{y^{-1}}\right]^{-2} \left[\frac{x^{-5}}{y^5}\right]^3$. Record your answer with positive exponents. (3 marks)

$$= \left(\frac{y^{-1}}{x^{-4}}\right)^2 \left(\frac{x^{-5}}{y^5}\right)^3$$

$$= \frac{y^{-2}}{x^{-8}} \times \frac{x^{-15}}{y^{15}}$$

$$= x^{-7} y^{-17}$$

$$= \frac{1}{x^7 y^{17}}$$

21. Determine the roots of $3x^2 - 12x + 5 = 0$. If necessary, leave your answer in simplified radical form. *(no decimals)* (4 marks)

$$x = \frac{12 \pm \sqrt{(-12)^2 - 4(3)(5)}}{2(3)}$$

$$= \frac{12 \pm \sqrt{144 - 60}}{6}$$

$$= \frac{12 \pm \sqrt{84}}{6}$$

$$= \frac{12 \pm 2\sqrt{21}}{6}$$

$$= \frac{6 \pm \sqrt{21}}{3}$$

FORMULAS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$t_n = ar^{n-1}$$

$$S_n = \frac{n}{2}(t_1 + t_n)$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$A = P(1+i)^n \quad PV = A(1+i)^{-n}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}, \quad r \neq 1$$

$$FV = PMT \left(\frac{(1+i)^n - 1}{i} \right)$$

$$PV = PMT \left(\frac{1 - (1+i)^{-n}}{i} \right)$$

Trigonometric Identities

$$\csc \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\cot \theta = \frac{1}{\tan \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$\cot^2 \theta + 1 = \csc^2 \theta$$